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## $D \rightarrow a_1, f_1$ transition form factors and semileptonic decays via 3-point QCD sum rules

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By using the 3-point QCD sum rules, we calculate the transition form factors of  $D$  decays into the spin triplet axial vector mesons  $a_1(1260)$ ,  $f_1(1285)$ ,  $f_1(1420)$ . In the calculations, we consider the quark contents of each meson in detail. In view of the fact that the isospin of  $a_1(1260)$  is one, we calculate the  $D^+ \rightarrow a_1^0(1260)$  and  $D^0 \rightarrow a_1^-(1260)$  transition form factors separately. In the case of  $f_1(1285)$ ,  $f_1(1420)$ , the mixing between light flavor  $SU(3)$  singlet and octet is taken into account. Based on the form factors obtained here, we give predictions for the branching ratios of relevant semileptonic decays, which can be tested in the future experiments.

*Keywords:*  $D$  meson; axial vector mesons; form factors; 3-point QCD sum rules.

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### 1. Introduction

The heavy to light meson exclusive decays are very important in testing the particle physics standard model (SM), extracting its parameters and searching for possible new physics. So far, the decays with only S-wave mesons (pseudoscalar mesons, vector mesons etc.) in the final states have been analyzed extensively both from theoretical and experimental aspects. Comparatively speaking, few studies have been done on those decays involving P-wave mesons (scalar mesons, axial vector mesons etc.) in the final states. In the last ten years, with the development of experiments, a large amount of decays with P-wave mesons in the final states have been found and measured,<sup>1-7</sup> which promote the theoretical investigations on these decays. To give predictions for the exclusive decays, one needs the knowledge of

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transition form factors. K.C. Yang et al. gave the form factors of  $B_{(s)}$  to P-wave mesons by using the light cone sum rules and studied relevant exclusive decays.<sup>8–10</sup>

A special role is played by the  $D_{(s)}$  exclusive decays, which provide an unique place to probe the new physics coupling with up type quarks. However, the light cone sum rules adopted in Ref.<sup>8–10</sup> are based on an expansion over  $m_A/m_b$  ( $m_A$  and  $m_b$  denote the mass of axial vector meson and bottom quark respectively) and therefore can not be used to calculate the form factors of  $D$  to axial vector mesons. R. Khosravi et al. calculated the  $D_{(s)}$  to  $K_1$  transition form factors in 3-point QCD sum rules.<sup>11</sup> In the present paper, we intend to calculate the semileptonic form factors of  $D$  to spin triplet axial vector mesons  $a_1(1260)$ ,  $f_1(1285)$ ,  $f_1(1420)$  by using the 3-point QCD sum rules and give a prediction for the branching ratios of relevant semileptonic decays.

## 2. Definitions and expressions of $D$ to axial vector meson transition form factors via 3-point QCD sum rules

Following Ref.,<sup>11</sup> the semileptonic  $D$  to axial vector meson  $A$  transition form factors are defined via corresponding hadronic matrix elements as follows,

$$\langle A(p', \varepsilon) | \bar{d} \gamma_\mu \gamma_5 c | D(p) \rangle = -\frac{2f_V(q^2)}{(m_D + m_A)} \epsilon_{\mu\nu\alpha\beta} \varepsilon^\nu p^\alpha p'^\beta \quad (1)$$

$$\begin{aligned} \langle A(p', \varepsilon) | \bar{d} \gamma_\mu c | D(p) \rangle = i \left[ f_0(q^2)(m_D + m_A) \varepsilon_\mu - \frac{f_1(q^2)}{(m_D + m_A)} \varepsilon \cdot p P_\mu \right. \\ \left. - \frac{f_2(q^2)}{(m_D + m_A)} \varepsilon \cdot p q_\mu \right] \quad (2) \end{aligned}$$

Here  $A$  denotes the axial vector meson  $a_1$  or  $f_1$ .  $P_\mu = (p + p')_\mu$ ,  $q_\mu = (p - p')_\mu$  and  $\varepsilon$  is the polarization vector of  $A$ .

The form factors  $f_V$ ,  $f_0$ ,  $f_1$ ,  $f_2$  can be evaluated via 3-point QCD sum rules. As stated in Ref.,<sup>11</sup> the following redefinitions are needed in order for the calculations to be simple,

$$\begin{aligned} F_V(q^2) &= \frac{2f_V(q^2)}{m_D + m_A}, & F_0(q^2) &= (m_D + m_A) f_0(q^2) \\ F_1(q^2) &= -\frac{f_1(q^2)}{m_D + m_A}, & F_2(q^2) &= -\frac{f_2(q^2)}{m_D + m_A} \end{aligned} \quad (3)$$

Using the same analysis as Ref.<sup>11</sup> with simply choosing  $D_q$  to be  $D$  and replacing  $K_{1A(B)}$  with  $A$ , we have

$$\begin{aligned} F_i(q^2, s_0, s'_0, M_1^2, M_2^2) &= -\frac{(m_c + m_q)}{f_D m_D^2 f_A m_A} e^{(m_D^2/M_1^2)} e^{(m_A^2/M_2^2)} \left\{ -\frac{1}{4\pi^2} \int_{m_c^2}^{s'_0} ds' \right. \\ &\quad \times \int_{s_L}^{s_0} ds \rho_i(s, s', q^2) e^{(-s/M_1^2)} e^{(-s'/M_2^2)} \\ &\quad \left. + M_1^2 M_2^2 \mathcal{B}_{p^2}(M_1^2) \mathcal{B}_{p'^2}(M_2^2) \Pi_i^{\text{non-per}}(p^2, p'^2, q^2) \right\} \quad (4) \end{aligned}$$

where  $i = V, 0, 1, 2$ . The expressions of spetral densities  $\rho_i(s, s', q^2)$  and corresponding nonperturbative contributions  $\Pi_i^{\text{non-per}}(p^2, p'^2, q^2)$  can be found in Ref.<sup>11</sup>  $s_0$  and  $s'_0$  are the continuum thresholds in  $D$  and axial vector meson  $A$  channels respectively. The lower limit  $s_L$  in the integration over  $s$  is

$$s_L = \frac{(m_q^2 + q^2 - m_c^2 - s')(m_c^2 s' - m_q^2 q^2)}{(m_c^2 - q^2)(m_c^2 - s')} \quad (5)$$

### 3. Numerical results and analysis of the form factors

With the sum rule expressions above, it is now in the position to evaluate the  $D$  to axial vector meson transition form factors. According to the quark model,<sup>1</sup> the flavor contents of relevant mesons are

$$\begin{aligned} D^+ &= c\bar{d}, & D^0 &= c\bar{u}, & a_1^0(1260) &= \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}), & a_1^-(1260) &= \bar{u}d, \\ f_1 &= \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s}), & f_8 &= \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} - 2s\bar{s}) \end{aligned} \quad (6)$$

Note that the physical  $f_1(1285), f_1(1420)$  states are mixtures of light flavor  $SU(3)$  singlet  $f_1$  and octet  $f_8$ , which can be expressed as

$$f_1(1285) = f_1 \cos \theta + f_8 \sin \theta, \quad f_1(1420) = -f_1 \sin \theta + f_8 \cos \theta \quad (7)$$

For the mixing angle  $\theta$ , we choose the value  $\theta = 23^\circ$  given by H.Y.Cheng based on phenomenological analysis.<sup>12</sup> The masses and decay constants of relevant mesons are collected in Table1. Other input parameters are quark masses and parameters relating to condensates, which are listed as

$$\begin{aligned} m_c &= 1.27\text{GeV}, & m_d &= 0.005\text{GeV}, \\ \langle d\bar{d} \rangle &= -1.38 \times 10^{-2}\text{GeV}^3, & m_0^2 &= 0.8\text{GeV}^2 \end{aligned}$$

Table 1. Masses and decay constants of relevant mesons. The masses of  $D^+, D^-, a_1(1260)$  are experimental values given by the Particle Data Group(PDG),<sup>1</sup> while other quantities are obtained in the QCD sum rule calculation.<sup>13</sup>

State	Mass [GeV]	Decay constant $f$ [GeV]
$D^+$	1.87	0.222
$D^-$	1.86	0.222
$a_1(1260)$	1.26	0.238
$f_1$	1.28	0.245
$f_8$	1.29	0.239

From Eq.(4), it is easily seen that the sum rules for form factors contain four free parameters  $M_1^2, M_2^2, s_0, s'_0$ .  $M_1^2$  and  $M_2^2$  are Borel mass squares.  $s_0$  and  $s'_0$  are the

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continuum thresholds of  $D$  and  $A$  mesons respectively. These are not physical quantities, so the form factors as physical quantities should be independent of them. Practically, the allowed regions for  $M_1^2$ ,  $M_2^2$ ,  $s_0$ ,  $s'_0$  are determined by requiring the curves of form factors as functions of these four parameters to be most stable. As illustrations, we give the  $D^+ \rightarrow a_1^0(1260)$  transition form factors at zero momentum transfer ( $q^2 = 0$ ) as functions of Borel mass squares  $M_1^2$  in Fig.1. The corresponding variation of form factors with  $M_2^2$  and those for  $D^0 \rightarrow a_1^-(1260)$ ,  $D^+ \rightarrow f_1(1285)$ ,  $f_1(1420)$  decays are similar and shown in Fig.2-8.

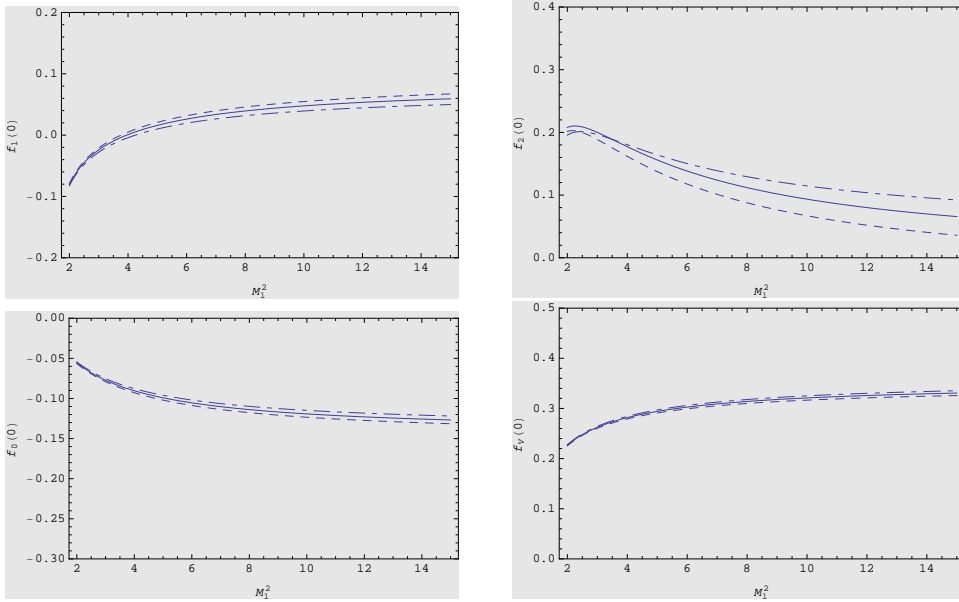


Fig. 1.  $D^+ \rightarrow a_1^0(1260)$  transition form factors at  $q^2 = 0$  as functions of  $M_1^2$ . The dashed, solid and dot dashed lines correspond to  $s_0 = 7.2$ ,  $7.0$  and  $6.8 \text{ GeV}^2$  respectively.

It is easily seen from Fig.1-8 that the form factors  $f_1$  and  $f_V$  are more insensitive to the  $s'_0$  and  $s_0$  respectively. Concretely, we choose the free parameters  $s_0 = 7.0 \pm 0.2 \text{ GeV}^2$ ,  $s'_0 = 3.5 \pm 0.2 \text{ GeV}^2$ ,  $M_1^2 = 8.0 \pm 0.5 \text{ GeV}^2$ ,  $M_2^2 = 7.0 \pm 0.5 \text{ GeV}^2$  in our calculations. Note that the sum rules become unreliable in the large  $q^2$  region and therefore our predictions for the form factors via 3-point sum rules are truncated at about  $q^2 = 0.15 \text{ GeV}^2$ . To extend the results to the whole kinematically allowed region, i.e.  $0 \leq q^2 \leq (m_D - m_A)^2$ , we use the following parametrization,

$$F(q^2) = \frac{F(0)}{1 - a(q^2/m_D^2) + b(q^2/m_D^2)^2} \quad (8)$$

Here  $F$  denotes the form factor  $f_1, f_2, f_0, f_V$ . By using this parametrization and the form factors at low  $q^2$  region calculated via 3-point sum rules, we obtain the form factors in the whole kinematically allowed region shown in Fig.9-12. The  $D^+ \rightarrow$

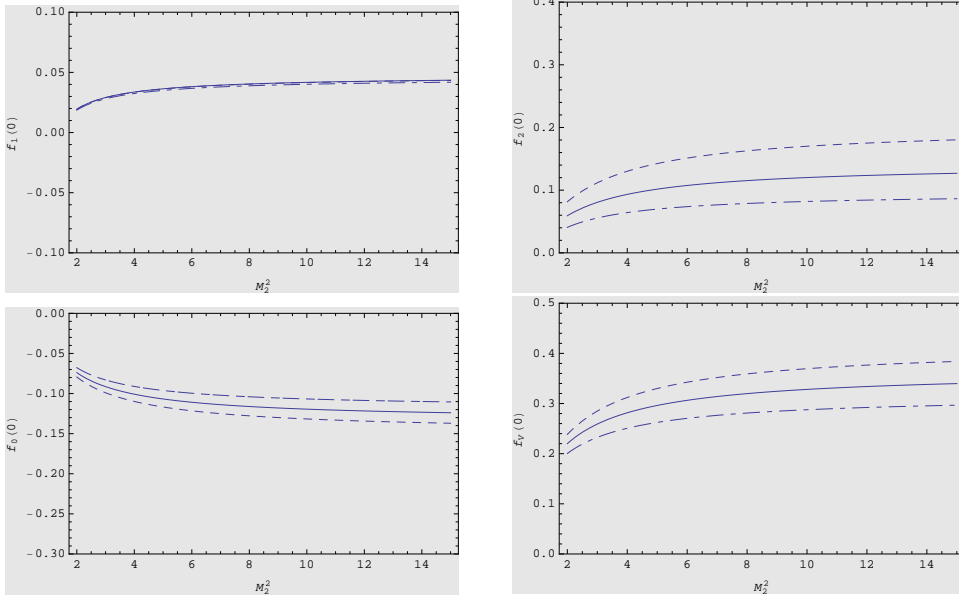


Fig. 2.  $D^+ \rightarrow a_1^0(1260)$  transition form factors at  $q^2 = 0$  as functions of  $M_2^2$ . The dashed, solid and dot dashed lines correspond to  $s'_0 = 3.7, 3.5$  and  $3.3 \text{ GeV}^2$  respectively.

$a_1^0(1260), D^0 \rightarrow a_1^-(1260), D^+ \rightarrow f_1(1285), f_1(1420)$  transition form factors at  $q^2 = 0$  and corresponding extrapolative parameters  $a, b$  are exhibited in Table 2, where the theoretical errors for  $a, b$  are not shown for simplicity.

Roughly speaking, the uncertainties of the form factors  $f_1, f_0$  and  $f_V$  are about (15-20)%, while that of  $f_2$  is around 30% which can be attributed to the large dependence of this form factor on the free parameters  $s_0, s'_0, M_1^2, M_2^2$ . In addition, the  $D^+ \rightarrow a_1^0(1260)$  transition form factors have opposite signs compared with those of other channels. This is because these decays are governed by the  $c \rightarrow d$  transition in the quark level and the flavor content of  $a_1^0(1260)$  is  $1/\sqrt{2}(u\bar{u} - d\bar{d})$  where the sign of  $d\bar{d}$  component is minus. Moreover, the magnitude of form factor  $f_1$  is obviously smaller than other form factors and the form factor  $f_0$  are almost independent of  $q^2$  for all channels we investigated. For the  $D^+ \rightarrow f_1(1285), f_1(1420)$  decays in which mixing is involved, the form factors of  $D^+ \rightarrow f_1(1285)$  are about five times larger than those of  $D^+ \rightarrow f_1(1420)$ , which is due to the fact that the flavor contents of  $f_1(1285)$  and  $f_1(1420)$  are dominated by the components  $1/\sqrt{2}(u\bar{u} + d\bar{d})$  and  $s\bar{s}$  respectively. The extrapolative parameters  $a, b$  are the same for  $D^+ \rightarrow a_1^0(1260), f_1(1285), f_1(1420)$  decays and this means the variations of corresponding transition form factors in these three channels with the momentum transfer squared  $q^2$  are similar.

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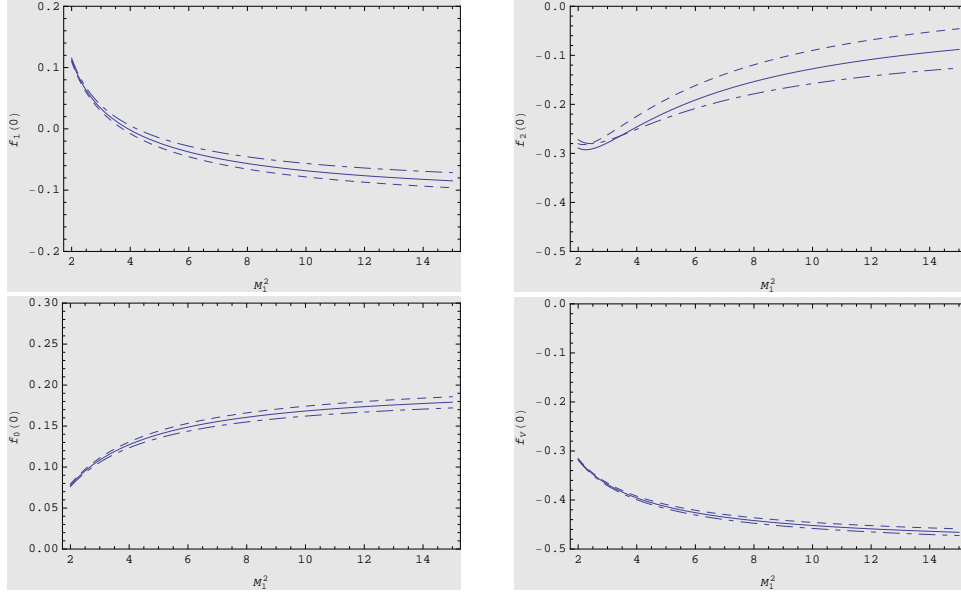


Fig. 3.  $D^0 \rightarrow a_1^-(1260)$  transition form factors at  $q^2 = 0$  as functions of  $M_1^2$ . The dashed, solid and dot dashed lines correspond to  $s_0 = 7.2, 7.0$  and  $6.8 \text{ GeV}^2$  respectively.

Table 2. The  $D^+ \rightarrow a_1^0(1260)$ ,  $D^0 \rightarrow a_1^-(1260)$ ,  $D^+ \rightarrow f_1(1285)$ ,  $f_1(1420)$  transition form factors at  $q^2 = 0$  and corresponding extrapolative parameters  $a, b$ .

Decay		$F(0)$	a	b
$D^+ \rightarrow a_1^0(1260)$	$f_1$	$0.039^{+0.012}_{-0.010}$	7.18	20.60
	$f_2$	$0.112^{+0.037}_{-0.032}$	-0.68	40.47
	$f_0$	$-0.114^{+0.018}_{-0.019}$	0.08	2.47
	$f_V$	$0.314^{+0.048}_{-0.046}$	2.04	-1.00
$D^0 \rightarrow a_1^-(1260)$	$f_1$	$-0.057^{+0.015}_{-0.016}$	7.04	19.65
	$f_2$	$-0.154^{+0.045}_{-0.052}$	-0.72	42.52
	$f_0$	$0.161^{+0.027}_{-0.025}$	0.08	2.49
	$f_V$	$-0.442^{+0.064}_{-0.068}$	2.03	-0.98
$D^+ \rightarrow f_1(1285)$	$f_1$	$-0.038^{+0.010}_{-0.010}$	7.18	20.60
	$f_2$	$-0.107^{+0.030}_{-0.035}$	-0.68	40.47
	$f_0$	$0.107^{+0.018}_{-0.016}$	0.08	2.47
	$f_V$	$-0.300^{+0.043}_{-0.047}$	2.04	-1.00
$D^+ \rightarrow f_1(1420)$	$f_1$	$-0.008^{+0.002}_{-0.002}$	7.18	20.60
	$f_2$	$-0.023^{+0.007}_{-0.008}$	-0.68	40.47
	$f_0$	$0.022^{+0.004}_{-0.003}$	0.08	2.47
	$f_V$	$-0.066^{+0.009}_{-0.010}$	2.04	-1.00

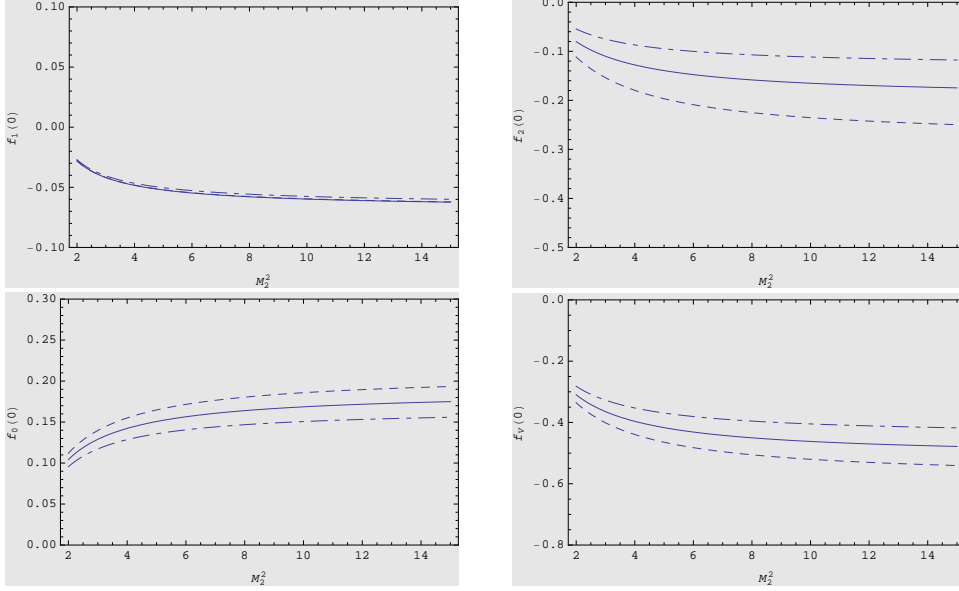


Fig. 4.  $D^0 \rightarrow a_1^-(1260)$  transition form factors at  $q^2 = 0$  as functions of  $M_2^2$ . The dashed, solid and dot dashed lines correspond to  $s'_0 = 3.7, 3.5$  and  $3.3 \text{ GeV}^2$  respectively.

#### 4. Branching ratios for semileptonic decays

In this section, we shall use the above obtained transition form factors to calculate the branching ratios of relevant semileptonic decays. Specially, the decay channels we calculate are  $D^+ \rightarrow a_1^0(1260)l^+\nu_l$ ,  $D^0 \rightarrow a_1^-(1260)l^+\nu_l$ ,  $D^+ \rightarrow f_1(1285)l^+\nu_l$  and  $D^+ \rightarrow f_1(1420)l^+\nu_l$ . The lepton  $l$  may be an electron or a muon.

The differential decay widths for the process  $D \rightarrow Al^+\nu_l$  can be written as

$$\frac{d\Gamma}{dq^2} = \frac{d\Gamma_T}{dq^2} + \frac{d\Gamma_L}{dq^2} \quad (9)$$

where  $\frac{d\Gamma_T}{dq^2}$  and  $\frac{d\Gamma_L}{dq^2}$  denote the transverse and longitudinal differential decay width respectively. Neglecting the mass of lepton  $l$ , their expressions have the following forms,

$$\frac{d\Gamma_T}{dq^2} = \frac{G_F^2 |V_{cd}|^2}{192\pi^3 m_D^3} q^2 \lambda^{1/2}(m_D^2, m_A^2, q^2) (|H_+|^2 + |H_-|^2) \quad (10)$$

$$\frac{d\Gamma_L}{dq^2} = \frac{G_F^2 |V_{cd}|^2}{192\pi^3 m_D^3} \lambda^{1/2}(m_D^2, m_A^2, q^2) |H_0|^2 \quad (11)$$

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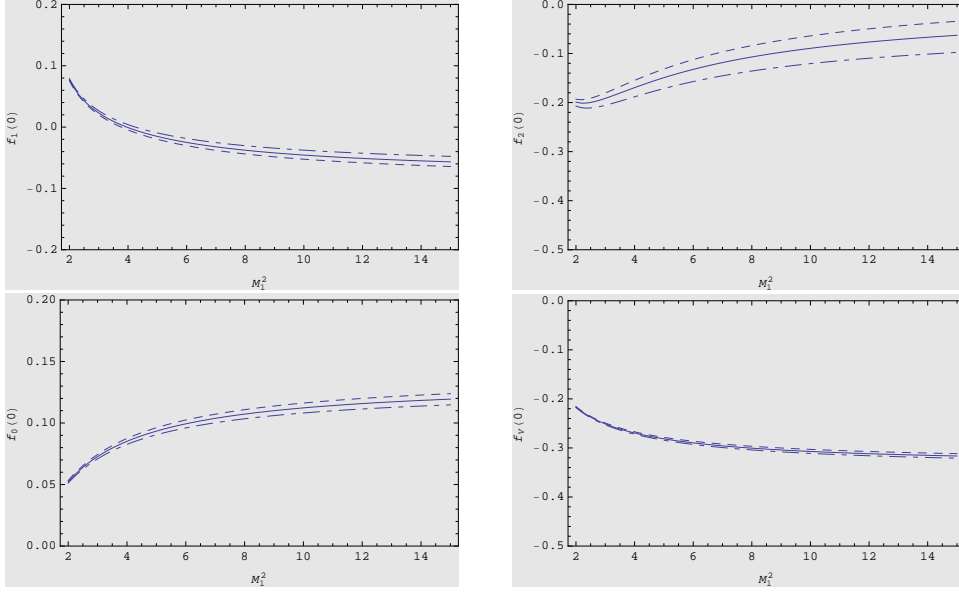


Fig. 5.  $D^+ \rightarrow f_1(1285)$  transition form factors at  $q^2 = 0$  as functions of  $M_1^2$ . The dashed, solid and dot dashed lines correspond to  $s_0 = 7.2, 7.0$  and  $6.8 \text{ GeV}^2$  respectively.

with

$$\begin{aligned}
 H_{\pm}(q^2) &= (m_D + m_K) f_0(q^2) \mp \frac{\lambda^{1/2}(m_D^2, m_A^2, q^2)}{m_D + m_A} f_V(q^2) \\
 H_0(q^2) &= \frac{1}{2m_A} \left[ (m_D^2 - m_A^2 - q^2)(m_D + m_A) f_0(q^2) \right. \\
 &\quad \left. - \frac{\lambda^{1/2}(m_D^2, m_A^2, q^2)}{m_D + m_A} f_1(q^2) \right] \\
 \lambda(m_D^2, m_A^2, q^2) &= m_D^2 + m_A^2 + q^4 - 2m_D^2 q^2 - 2m_A^2 q^2 - 2m_D^2 m_A^2
 \end{aligned}$$

For the CKM matrix element  $|V_{cd}|$  and Fermi coupling constant  $G_F$ , we use the latest values given by PDG, i.e.  $|V_{cd}| = 0.225 \pm 0.008$ ,  $G_F = 1.116 \times 10^{-5} \text{ GeV}^{-2}$ . With the above considerations, we obtain the differential decay widths as functions of  $q^2$  for  $D^+ \rightarrow a_1^0(1260)l^+\nu_l$ ,  $D^0 \rightarrow a_1^-(1260)l^+\nu_l$ ,  $D^+ \rightarrow f_1(1285)l^+\nu_l$  and  $D^+ \rightarrow f_1(1420)l^+\nu_l$  decays, the center values of which are shown in Fig.13.

From Fig.13, we can observe that the differential decay widths for  $D^+ \rightarrow a_1^0(1260)l^+\nu_l$  are about half of those for  $D^0 \rightarrow a_1^-(1260)l^+\nu_l$  which can be ascribed to the factor  $1/\sqrt{2}$  in the flavor content of  $a_1^0$  (see Eq.(6)). In addition, the differential decay widths for  $D^+ \rightarrow f_1(1420)l^+\nu_l$  are almost two order of magnitude smaller than those for  $D^+ \rightarrow f_1(1285)l^+\nu_l$  and this is related to the fact that  $f_1(1420)$  is nearly a pure  $s\bar{s}$  state. The transverse differential decay width vanishes at the largest recoil, i.e.  $q^2 = 0$  point due to the factor  $q^2$  in the front of Eq.(10).



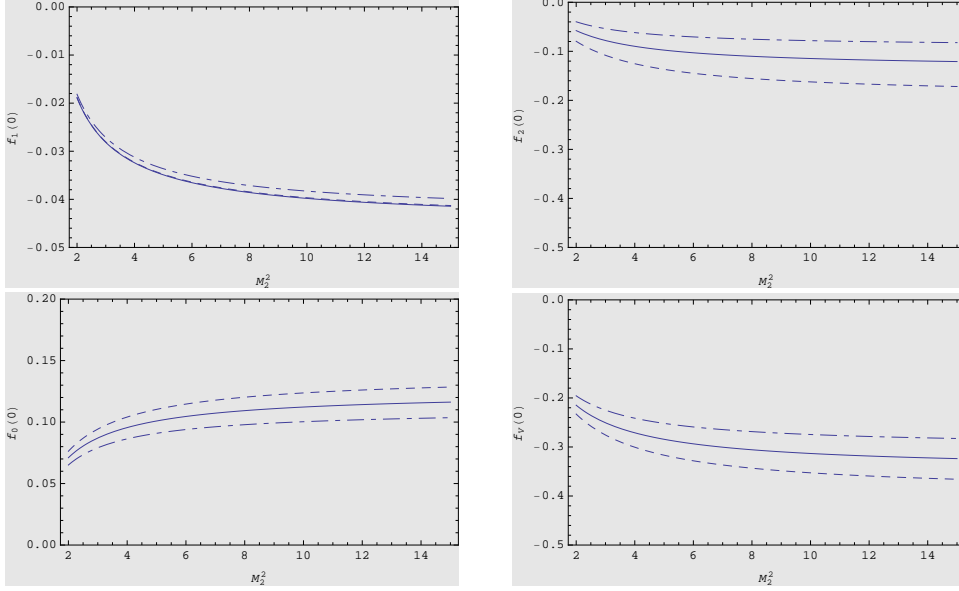


Fig. 6.  $D^+ \rightarrow f_1(1285)$  transition form factors at  $q^2 = 0$  as functions of  $M_2^2$ . The dashed, solid and dot dashed lines correspond to  $s'_0 = 3.7, 3.5$  and  $3.3 \text{ GeV}^2$  respectively.

Integrating the differential decay widths over  $q^2$  and using the lifetimes of  $D$  mesons given by PDG, i.e.  $\tau_{D^+} = 1.04 \times 10^{-12} \text{ s}$ ,  $\tau_{D^0} = 0.41 \times 10^{-12} \text{ s}$ , we get the following branching ratios for relevant semileptonic decays,

$$Br(D^+ \rightarrow a_1^0(1260)l^+\nu_l) = 1.47^{+0.44+0.11}_{-0.34-0.10} \times 10^{-5} \quad (12)$$

$$Br(D^0 \rightarrow a_1^-(1260)l^+\nu_l) = 1.11^{+0.33+0.08}_{-0.26-0.08} \times 10^{-5} \quad (13)$$

$$Br(D^+ \rightarrow f_1(1285)l^+\nu_l) = 1.07^{+0.32+0.07}_{-0.25-0.08} \times 10^{-5} \quad (14)$$

$$Br(D^+ \rightarrow f_1(1420)l^+\nu_l) = 1.22^{+0.39+0.09}_{-0.31-0.09} \times 10^{-7} \quad (15)$$

where the first and second uncertainties come from the transition form factors and CKM matrix element  $|V_{cd}|$  respectively. The uncertainties coming from transition form factors are about (20-30)% and those stemming from  $|V_{cd}|$  are around 7%. The branching ratios for  $D^+ \rightarrow a_1^0(1260)l^+\nu$ ,  $D^0 \rightarrow a_1^-(1260)l^+\nu_l$  and  $D^+ \rightarrow f_1(1285)l^+\nu$  are at the order of  $\mathcal{O}(10^{-5})$ , while the one for  $D^+ \rightarrow f_1(1420)l^+\nu$  is approximately two order of magnitude smaller which is similar to the differential decay width. Note that these branching ratios have not been measured, our results can be tested by the more precise experiments such as LHCb, BelleII, etc. in the future.

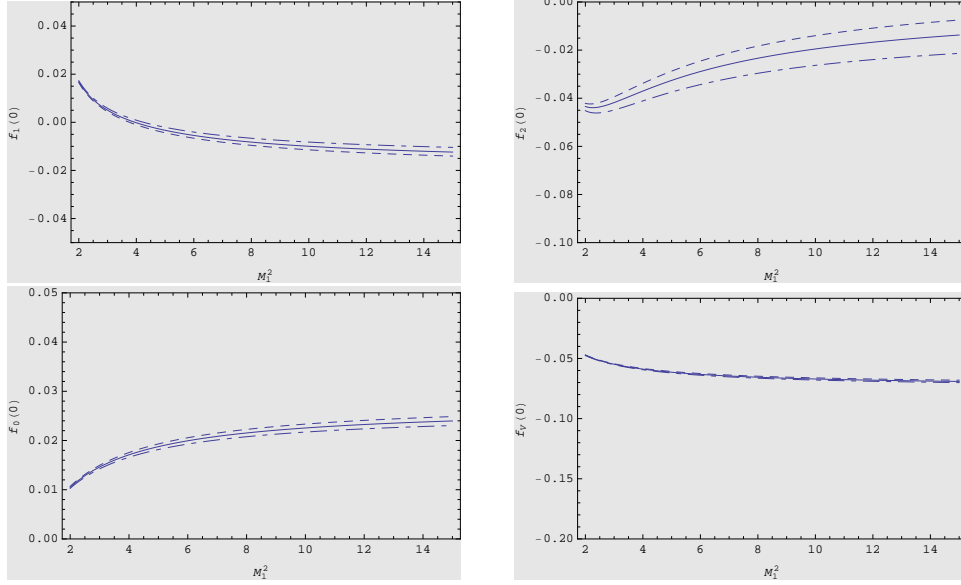
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Fig. 7.  $D^+ \rightarrow f_1(1420)$  transition form factors at  $q^2 = 0$  as functions of  $M_1^2$ . The dashed, solid and dot dashed lines correspond to  $s_0 = 7.2, 7.0$  and  $6.8 \text{ GeV}^2$  respectively.

## 5. Conclusion

In the present paper, we calculate the  $D$  to spin triplet axial vector meson  $a_1^0(1260)$ ,  $a_1^-(1260)$ ,  $f_1(1285)$ ,  $f_1(1420)$  transition form factors by using the 3-point QCD sum rules. The flavor contents of each meson and the mixing between  $f_1(1285)$  and  $f_1(1420)$  are considered in detail. The uncertainties of transition form factors comes from the four free parameters  $s_0, s'_0, M_1^2, M_2^2$  which is about (15-20)% for  $f_1, f_0, f_V$  and 30% for  $f_2$ . The transition form factors of  $D^+ \rightarrow f_1(1285)$  are about five times larger than those of  $D^+ \rightarrow f_1(1420)$  due to the fact that the flavor contents of  $f_1(1285)$  and  $f_1(1420)$  are dominated by the components  $1/\sqrt{2}(u\bar{u} + d\bar{d})$  and  $s\bar{s}$  respectively. Based on the form factors obtained here, we predict the branching ratios of relevant semileptonic decays. The branching ratios for  $D^+ \rightarrow a_1^0(1260)l^+\nu$ ,  $D^0 \rightarrow a_1^-l^+\nu_l$  and  $D^+ \rightarrow f_1(1285)l^+\nu$  are at the order of  $\mathcal{O}(10^{-5})$ , while the one for  $D^+ \rightarrow f_1(1420)l^+\nu$  is approximately two order of magnitude smaller. These results can be tested by the more precise experiments in the future.

## Acknowledgments

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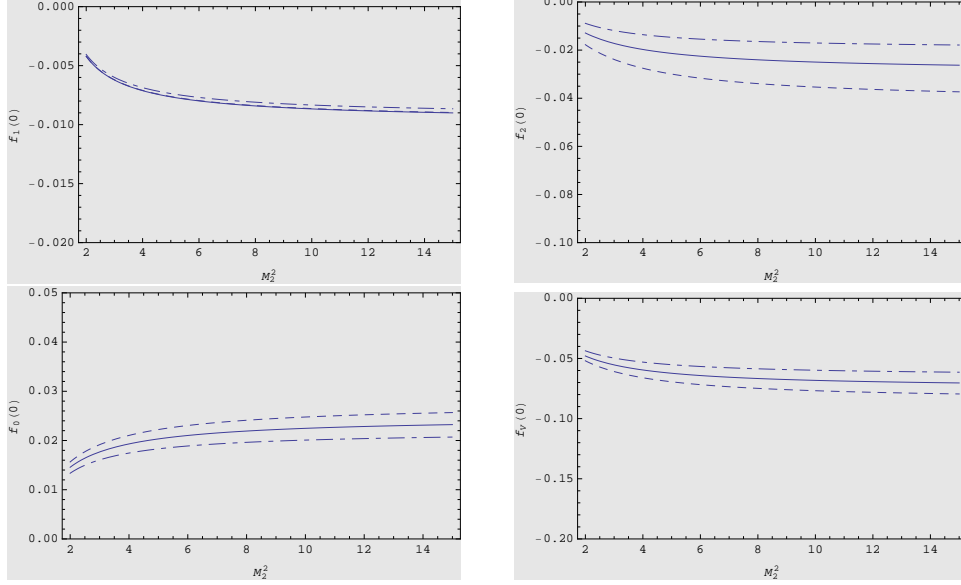


Fig. 8.  $D^+ \rightarrow f_1(1420)$  transition form factors at  $q^2 = 0$  as functions of  $M_2^2$ . The dashed, solid and dot dashed lines correspond to  $s'_0 = 3.7, 3.5$  and  $3.3 \text{ GeV}^2$  respectively.

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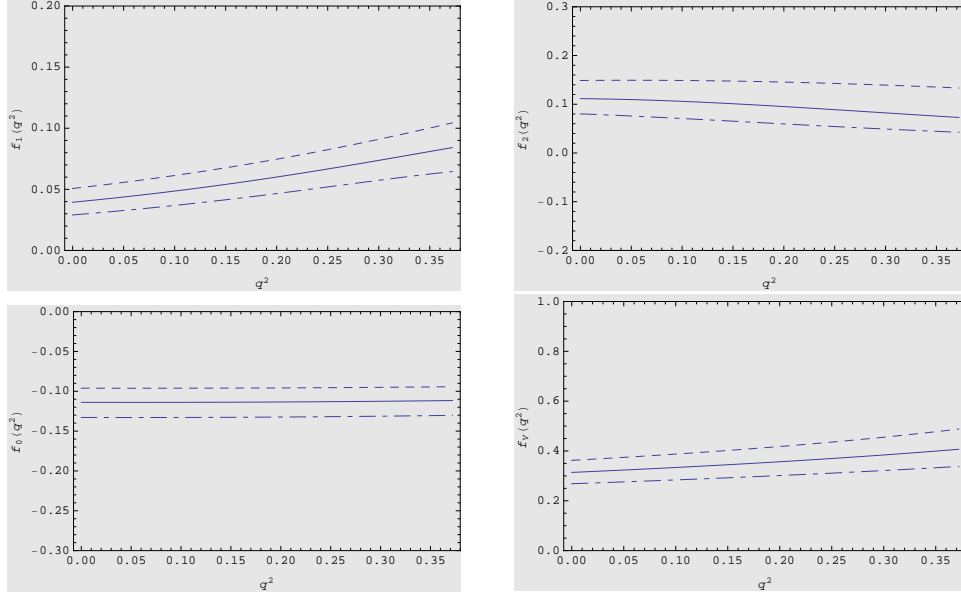


Fig. 9. The variation of  $D^+ \rightarrow a_1^0(1260)$  transition form factors as functions of  $q^2$ . The solid, dot dashed and dashed lines correspond to the center value, upper and lower limit respectively.

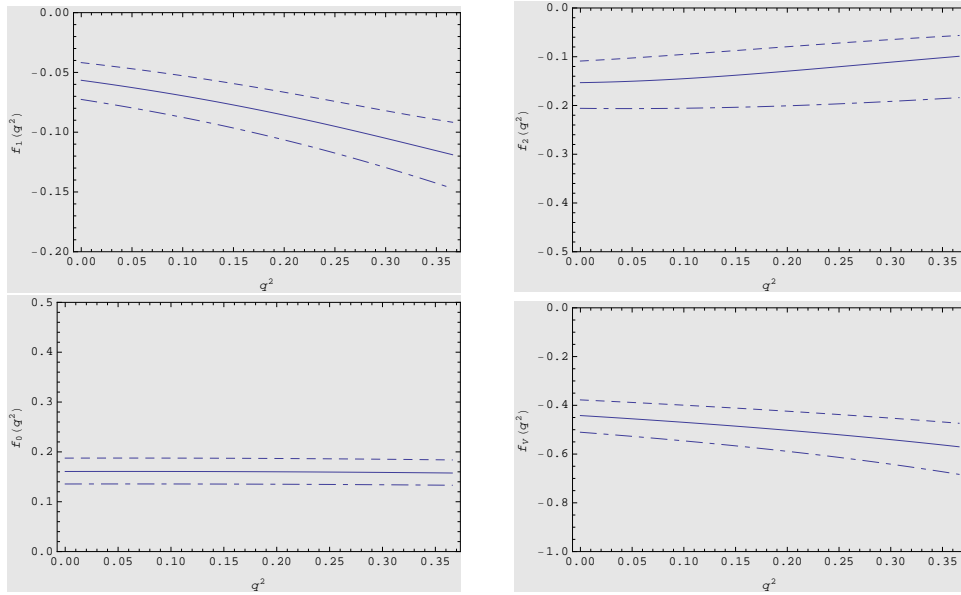


Fig. 10. The variation of  $D^0 \rightarrow a_1^-(1260)$  transition form factors as functions of  $q^2$ . The solid, dot dashed and dashed lines correspond to the center value, upper and lower limit respectively.

$D \rightarrow a_1, f_1$  transition form factors and semileptonic decays via 3-point QCD sum rules 13

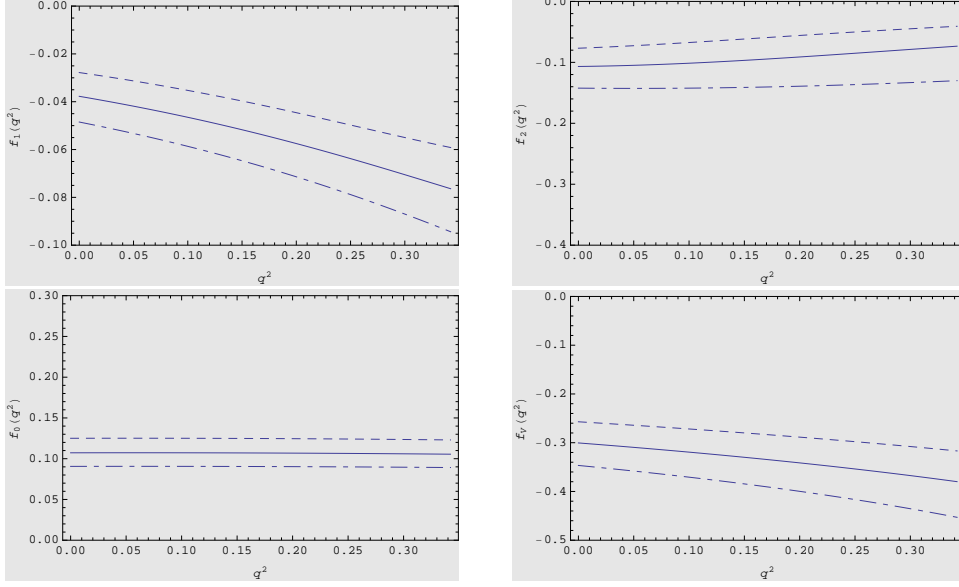


Fig. 11. The variation of  $D^+ \rightarrow f_1(1285)$  transition form factors as functions of  $q^2$ . The solid, dot dashed and dashed lines correspond to the center value, upper and lower limit respectively.

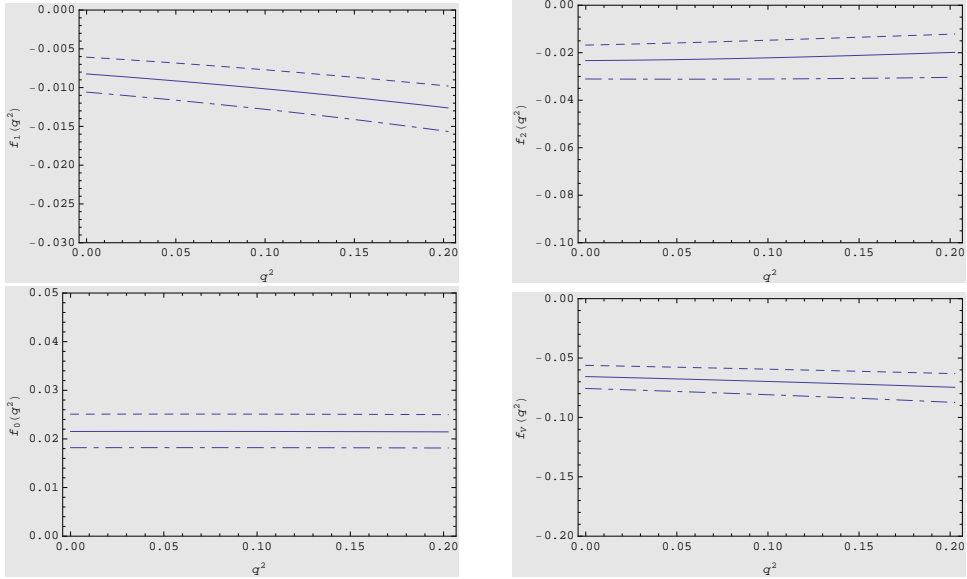


Fig. 12. The variation of  $D^+ \rightarrow f_1(1420)$  transition form factors as functions of  $q^2$ . The solid, dot dashed and dashed lines correspond to the center value, upper and lower limit respectively.

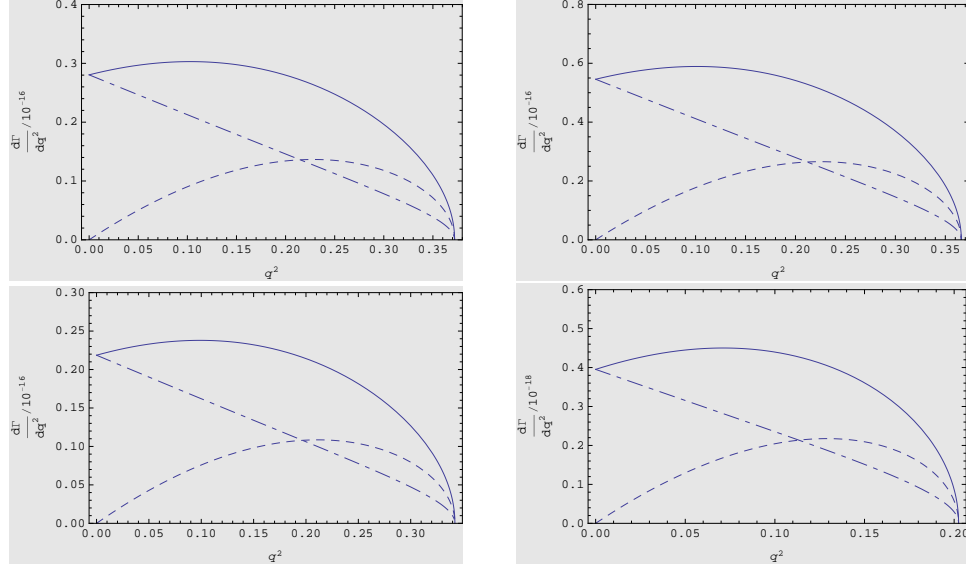
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Fig. 13. The differential decay widths of  $D^+ \rightarrow a_1^0(1260)l^+\nu_l$ ,  $D^0 \rightarrow a_1^-(1260)l^+\nu_l$ ,  $D^+ \rightarrow f_1(1285)l^+\nu_l$  and  $D^+ \rightarrow f_1(1420)l^+\nu_l$  as functions of  $q^2$ , which are shown in the upper-left, upper-right, lower-left and lower-right plots respectively. The dashed, dot dashed and solid lines correspond to the transverse, longitudinal and total differential decay width respectively.